TUTORIAL SHEET-1

RANK OF MATRIX

1. Find rank of minor method, A=$\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$
2. Find the value(s) of λ for which the rank of the matrix

$$A=\begin{bmatrix} \lambda & 1 & 0 \\ 1 & \lambda & 1 \\ 0 & 1 & \lambda \end{bmatrix}$$

is less than 3.

3. Find the value(s) of k so that the rank of the matrix

$$A=\begin{bmatrix} 2 & -1 & 3 \\ 8 & 2 & 0 \\ 0 & 1 & k \end{bmatrix}$$

is less than 3. Also find the rank for these values of k.

4. Find the value(s) of x such that the rank of the matrix

$$A=\begin{bmatrix} x+a & a & a \\ b & x+b & b \\ c & c & x+c \end{bmatrix}$$

is less than 3.

5. Find the rank of Matrix by using echelon form A=$\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$
6. Reduce the matrix into its echelon form and hence find its rank.

$$A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

7. Reduce the matrix to normal form and hence find their rank.

$$A=\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

8. Reduce the matrix to normal form and hence find their rank.

$$A=\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

9. Reduce to normal form and hence find its rank.

$$A=\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

TUTORIAL SHEET-2

GAUSS JORDAN METHOD, L.I./L.D. VECTORS

1. Use Gauss Jordan method to find the inverse of the matrix.

$$A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$$

2. Find the inverse of the matrix using Gauss Jordan method

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 7\\ 3 & 1 & 2\end{bmatrix}$$

3. Determine whether the set of vectors $(2,-1,0),(1,1,-1),(3,0,-1)$ is linearly dependent or independent.

4. Check if the vectors $(1,0,1,2),(2,1,3,4),(3,1,4,6)$ are linearly dependent or independent.

5. Are the vectors $X_1=(3,-2,4)$, $X_2=(1,0,2)$,$X_3=(1,-1,1)$ L.D.? If so, express one of these as a linear combination of the following.

6. For which values of k, the vectors $(1,2,3),(2,-1,4),(k,5,6)$ are linearly dependent?

TUTORIAL SHEET-3

SYSTEM OF EQUATIONS

1. Solve the equations:

   $4x + 2y + z + 3w = 0$

   $6x + 3y + 4z + 7w = 0$

   $2x + y + w = 0$

2. Solve the equations:

   $5x + 3y + 7z = 4$

   $3x + 26y + 2z = 9$

   $7x + 2y + 11z = 5$

3. Find for what values of K, the set of equations

   $2x - 3y + 6z - 5t = 3$

   $y - 4z + t = 1$

   $4x - 5y + 8z - 9t = k$

   (a) No solution

   (b) Infinite no of solutions

   (c) Unique Solution

4. For what values of α & β do the system of equations

   $x + y + z = 6$

   $x + 2y + 3z = 10$

   $x + 2y + \alpha z = \beta$

   have

   (a) No Solution
   (b) Unique Solution
   (c) More than one solution

5. Discuss the consistency of the following system of equations:

   $x + y + z = 4$

   $2x + y + z = 5$

   $x - y + z = 0$

6. Solve

   $x+2y+z=3$
   $2x+3y+2z=5$
   $3x-5y+5z=2$
   $3x+9y-z=4.$

7. For what value of K will the following equation fail to have unique solution.

   $3x-y+Kz=1$
   $2x+y+z=2$
   $X+2y-Kz=-1$
   will the equation have any solution for these values of K?

8. Discuss the consistency of the following system of equations.

   $x+y-2z+3w=0$,
   $x-2y+z-w=0$,
   $4x+y-5z+8w=0$,
   $5x-7y+2z-w=0$.

9. Solve completely the system of equations:

   $3x+4y-z-6w=0;$
   $2x+3y+2z-3w=0;$
   $2x+y-14z-9w=0;$
   $x+3y+13z+3w=0.$

TUTORIAL SHEET-4

EIGEN VALUES/VECTORS, CAYLEY HAMILTON THEOREM

1. Find the eigen value and eigen vector of the matrix A
$$A=\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ 0 & 2 & 3\end{bmatrix}$$
2. Find the sum and product of eigen values of the matrix A
$$A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$$
3. If the eigenvalues of A are 1,2,4. Find value of k.
$$A=\begin{bmatrix}2 & -1 & 0\\ -1 & 3 & 1\\ 0 & 1 & k\end{bmatrix}$$
4. Find the latent roots of the matrix A^{-1}
$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$
5. Find the eigen value and eigen vector of the matrix A^2
$$A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$
6. Find the eigen values the matrix A-I and A+5I
$$A=\begin{bmatrix}1 & 2 & 2\\ 1 & 2 & -1\\ -1 & 1 & 4\end{bmatrix}$$
7. Verify the cayley Hamilton theorem for the matrix A and find A^{-1}
$A = \begin{bmatrix}3 & -7\\ 4 & 5\end{bmatrix}$,
8. Verify the cayley Hamilton theorem for the matrix A and find A^{-1}
$$A=\begin{bmatrix}0 & 0 & 1\\ 1 & 2 & 0\\ 2 & -1 & 0\end{bmatrix}$$
9. Using Cayley Hamilton theorem for
$A = \begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$, Find value of $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$.
10. Using Cayley Hamilton theorem for
$A = \begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$, Find A^8 & A^{10}.